Conics – Circles Notes

Locus/Loci

- a set of point that have a common metric property
  - ex: A circle is a locus of points all equal distance from the center
- a path of an object which can be defined by an equation
  - finding the equation of a locus amounts to finding a relation between x and y coordinates of point "P" that travel the locus

Conic

- Is the figure formed by the intersection of a plane with a conical surface
  - circle, ellipse, hyperbola, parabola
  - always a locus of points

Circle

A circle is a locus of points whereby each point is the same distance from the center. It is a relation (not function)

Basic Equation – centered at the origin, and where r is the radius

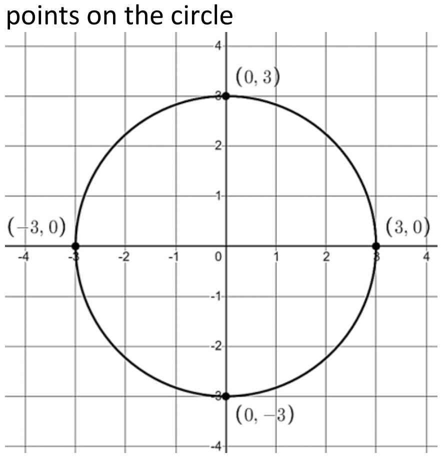
$$x^2 + y^2 = r^2$$

Transformed Equation – centered at (h,k), and where r is the radius

$$(x - h)^2 + (y - k)^2 = r^2$$

Ex: $x^2 + y^2 = 9$ and solve for y when $x = 2$

center at (0,0), radius of 3
From center, move right 3, left 3, up 3, and down 3 to get points on the circle

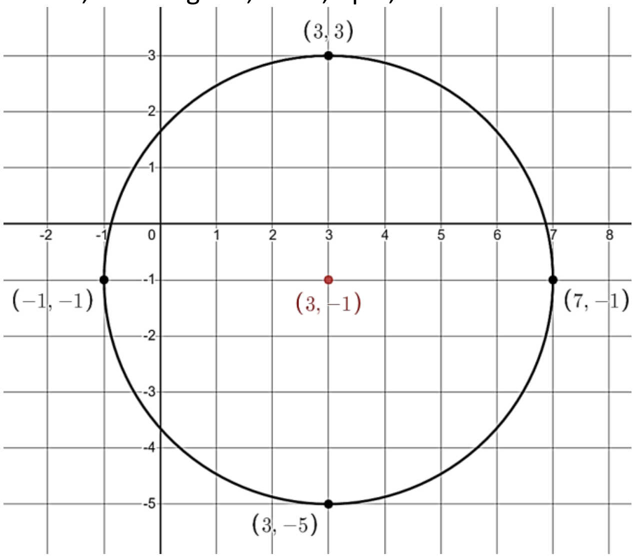


$$x^2 + y^2 = 9$$
$$2^2 + y^2 = 9$$
$$4 + y^2 = 9$$
$$y^2 = 5$$
$$y = \pm\sqrt{5}$$

*Note: unlike square root functions, we need to take both the positive and negative root when solving. Looking at the graph, when x is 2, we can see that there are 2 y-values, $+\sqrt{5}$ and $-\sqrt{5}$.

Ex: $(x-3)^2+(y+1)^2=16$ and solve for x when $y=2$

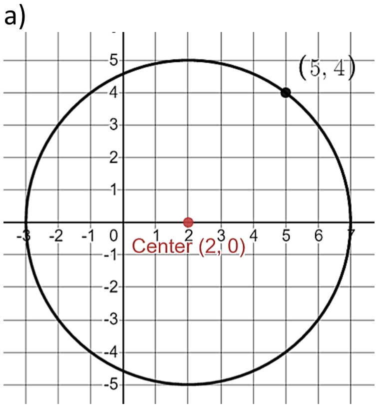
center at $(3,-1)$, radius of 4
From center, move right 4, left 4, up 4, down 4



$$(x-3)^2+(y+1)^2=16$$
$$(x-3)^2+(2+1)^2=16$$
$$(x-3)^2+(3)^2=16$$
$$(x-3)^2+9=16$$
$$(x-3)^2=7$$
$$(x-3)=+\sqrt{7}\ and\ (x-3)=-\sqrt{7}$$
$$x=3+\sqrt{7}\ and\ x=3-\sqrt{7}$$

Ex: Find the rule of the following circles in standard form (the basic or transformed equation above)

a)



Center is (2, 0), so
$$h=2\ and\ k=0$$
$$(x-h)^2+(y-k)^2=r^2$$
$$(x-2)^2+y^2=r^2$$

Use point (5, 4)
$$(5-2)^2+4^2=r^2$$
$$3^2+4^2=r^2$$
$$9+16=r^2$$
$$25=r^2$$

So
$$(x-2)^2+y^2=25$$

b)
$$x^2+6x+y^2-2y+1=0$$
$$x^2+6x+y^2-2y=-1$$

You will have to complete the square for x and for y.

Reminder: that the coefficient on x, divide by 2, and square it. Do the same for y. Add that to both sides.
$$x^2+6x+\mathbf{9}+y^2-2y+\mathbf{1}=-1+\mathbf{9}+\mathbf{1}$$
$$x^2+6x+9+y^2-2y+1=9$$

Now factor the x terms and the y terms
$$(x+3)^2+(y-1)^2=9$$

c)
$$x^2+y^2-8x+4y=-4$$
$$x^2-8x+y^2+4y=-4$$
$$x^2-8x+16+y^2+4y+4=16$$
$$(x-4)^2+(y+2)^2=16$$

Inequalities

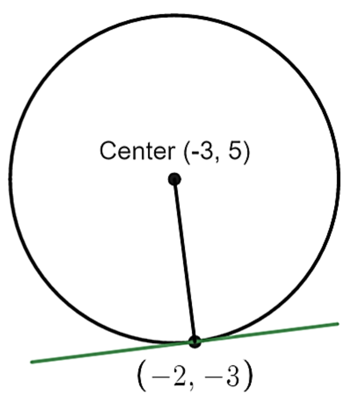
- if $(x-h)^2+(y-k)^2 \leq r^2$ use solid line and shade inside
- if $(x-h)^2+(y-k)^2 < r^2$ use dotted line and shade inside
- if $(x-h)^2+(y-k)^2 \geq r^2$ use solid line and shade outside
- if $(x-h)^2+(y-k)^2 \leq r^2$ use dotted line and shade outside

Tangent Lines

Line tangent to circle intersects (touches) the outside of the circle only once and is perpendicular to the radius.

Reminder: perpendicular lines have slopes that are negative reciprocals (flip the fraction and change the sign).

Ex: Find the equation of the tangent line drawn below.



Find slope of radius (going through the tangent point)

$$a=\frac{y_2-y_1}{x_2-x_1}$$

$$a=\frac{-3-5}{-2--3}=-\frac{8}{1}=-8$$

So slope of tangent line is the negative reciprocal, which is $\frac{1}{8}$

Find the equation of the line (using the known point on the line):

$$y=ax+b$$

$$y=\frac{1}{8}x+b$$

$$-3=\frac{1}{8}(-2)+b$$

$$-3=\frac{-1}{4}+b$$

$$-\frac{11}{4}=b$$

$$\therefore y=\frac{1}{8}x-\frac{11}{4}$$