Square Root Functions
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Properties of Radicals $\sqrt{a} = a^{\frac{1}{2}}$

really similar to exponent rules

1 PROPERTIES

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$ Example: $\sqrt[5]{a^2} = a^{\frac{2}{5}}$

$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$ " $\sqrt{7} \cdot \sqrt{5} = \sqrt{35}$

$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ " $\frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$

$m\sqrt{a} \cdot n\sqrt{b} = mn\sqrt{ab}$ " $2\sqrt{3} \cdot 4\sqrt{5} = 8\sqrt{15}$

$\frac{m\sqrt{ab}}{n\sqrt{b}} = \frac{m\sqrt{a}}{n}$ " $\frac{5\sqrt{3 \cdot 4}}{2\sqrt{4}} = \frac{5\sqrt{3}}{2}$

$\frac{m\sqrt{a} \cdot \cancel{\sqrt{b}}}{n\cancel{\sqrt{b}}}$

Breaking Apart Radicals

answer!

Ex. $\sqrt{48} = \sqrt{4} \cdot \sqrt{12} = 2\sqrt{12} = 2 \cdot \sqrt{3} \cdot \sqrt{4} = 2 \cdot \sqrt{3} \cdot 2 = 4\sqrt{3}$

$\frac{\sqrt{60}}{\sqrt{3}} = \sqrt{\frac{60}{3}} = \sqrt{20} \rightarrow \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$ ✓

Rationalizing the Denominator

Recall, you should not have a radical as a denominator.

$\frac{a}{\sqrt{b}}$ how do we remove this?

ex. $\frac{a}{\sqrt{b}} \cdot \left(\frac{\sqrt{b}}{\sqrt{b}}\right) = \frac{a\sqrt{b}}{b}$ ✓ $\left(b^{\frac{1}{2}}\right)^2$ $b^{\frac{2}{2}} = b^1$

same as 1

$a \cdot 2 \rightarrow a^2$ $2 \neq 4$ NOT

$$\cancel{\left(\frac{a}{\sqrt{b}}\right)^2 = \frac{a^2}{b}}$$

$\frac{2}{5} \neq \frac{4}{25}$ NOT equivalent

"1"

$\frac{2}{5}\left(\frac{2}{2}\right) = \frac{4}{10}$

ex. What if there is adding or subtracting in the denominator?

$$\frac{a}{\sqrt{b}+\sqrt{c}} \cdot \left(\frac{\sqrt{b}-\sqrt{c}}{\sqrt{b}-\sqrt{c}}\right) = \frac{a\sqrt{b}-a\sqrt{c}}{b-\cancel{\sqrt{bc}}+\cancel{\sqrt{bc}}-c}$$

$$= \frac{a\sqrt{b}-a\sqrt{c}}{b-c}$$

$\sqrt{b}\cdot\sqrt{b}$

$= (\sqrt{b})^2 = (b^{\frac{1}{2}})^2 = b$

Difference of squares

$(x+3)(x-3)$

$x^2 - \cancel{3x} + \cancel{3x} - 9$

$x^2 - 9$

62.

$$\boxed{\frac{9\sqrt[5]{160x^8y^{11}}}{3\sqrt[5]{5xy^2}}} = \frac{3}{1}\sqrt[5]{\frac{160x^8y^{11}}{5x^1y^2}}$$

$$= \frac{3}{1}\sqrt[5]{32x^7y^9}$$

$$= 3\sqrt[5]{32}\cdot\sqrt[5]{x^7}\cdot\sqrt[5]{y^9}$$

$$= 3\cdot 2\cdot\sqrt[5]{x^5}\cdot\sqrt[5]{x^2}\cdot\sqrt[5]{y^5}\cdot\sqrt[5]{y^4}$$

$$= 6xy\sqrt[5]{x^2y^4}$$

63.

$$\frac{2}{3+\sqrt{5}}\cdot\left(\frac{3-\sqrt{5}}{3-\sqrt{5}}\right) = \frac{6-2\sqrt{5}}{9-\cancel{3\sqrt{5}}+\cancel{3\sqrt{5}}-5} = \frac{6-2\sqrt{5}}{4}$$

reduce

$$= \frac{3-\sqrt{5}}{2}$$

Square Root Function

- inverse of a quadratic
- it's a semi-parabolic

RULE: $y = a\sqrt{b(x-h)} + k$ Vertex: (h,k)

→ or we can remove b (and you should) using properties of radicals

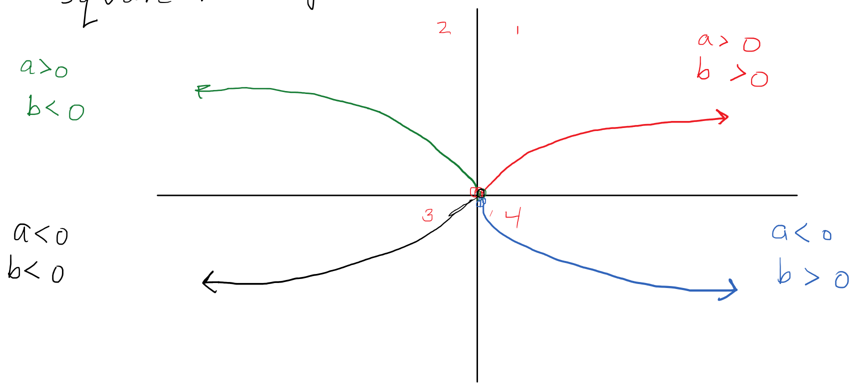
$$y = a\sqrt{\pm(x-h)} + k$$

Ex. Remove "b" term from the rule: $y = 2\sqrt{-9x+27} + 1$

$$y = 2\sqrt{-9(x-3)} + 1$$
$$y = 2 \cdot \sqrt{9} \cdot \sqrt{-(x-3)} + 1$$
$$y = (2)(3)\sqrt{-(x-3)} + 1$$
$$y = 6\sqrt{-(x-3)} + 1$$

There are four possibilities for direction and variation of the graph of a square root function.

$y = a\sqrt{b(x-h)} + k$

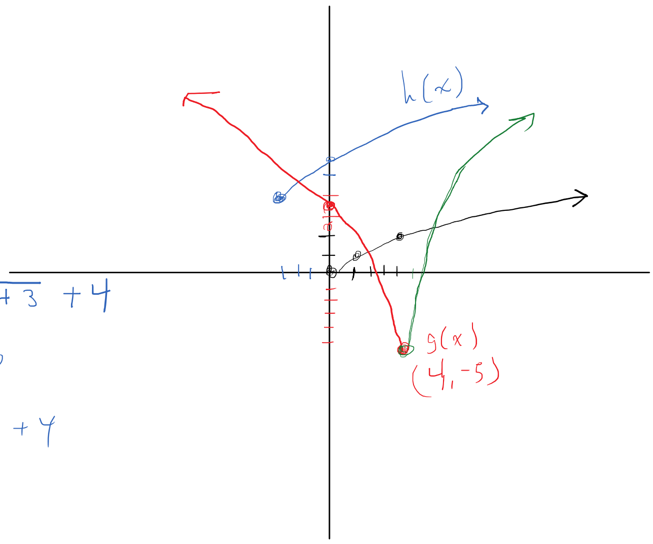


If $a<0$ then range is: $]-\infty, k]$

If $a>0$ then range is $[k, \infty[$

If $b<0$ then domain is: $]-\infty, h]$

If $b>0$ then domain is $[h, \infty[$



$h(x) = \sqrt{x+3} + 4$

y int, $x=0$

$= \sqrt{3} + 4$

$=$

Examples

$f(x) = \sqrt{x}$

$g(x) = 3\sqrt{-2(x-4)} - 5$

$a>0$
$b<0$

$i(x) = 3\sqrt{2x-8} - 5$

$= 3\sqrt{2(x-4)} - 5$

$a>0$
$b>0$

Finding the Rule

There are three steps to finding the rule of a square root function.

① Decide which rule applies:

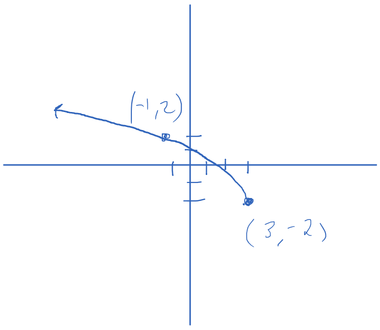
$y = a\sqrt{(x-h)} + k$ OR $y = a\sqrt{-(x-h)} + k$

*Hot tip: sketch the function to determine direction.

(2) Plug in vertex for (h,k)

(3) Plug in a given point (x,y) and solve for a.

Ex: Find the rule of a square root function with a vertex at $(3,-2)$ and passing through the point $(-1,2)$



$a>0$
$b<0$

$y = a\sqrt{-(x-h)} + k$

$y = a\sqrt{-(x-3)} - 2$

$2 = a\sqrt{-(-1-3)} - 2$

$2 = a\sqrt{-(-4)} - 2$

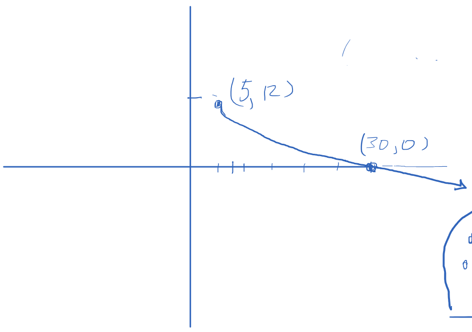
$4 = a\sqrt{4}$

$4 = 2a$

$\underline{2 = a}$

$\therefore\ y = 2\sqrt{-(x-3)} - 2$

Ex. Find the rule. Vertex $(5,12)$, and passes through $(30,0)$.



$a<0$
$b>0$

$y = a\sqrt{(x-h)} + k$

$y = a\sqrt{x-5} + 12$

$0 = a\sqrt{30-5} + 12$

$-12 = a\sqrt{25}$

$-12 = 5a$

$-\frac{12}{5} = a$

$\therefore\ y = -\frac{12}{5}\sqrt{(x-5)} + 12$

Graphing

Tips: - determine signs of a and b ($\therefore$ direction of the $f(x)$)
- create a table of values for additional points (choose x, solve y)

Graph the function: $y = -2\sqrt{-3x+12} - 2$

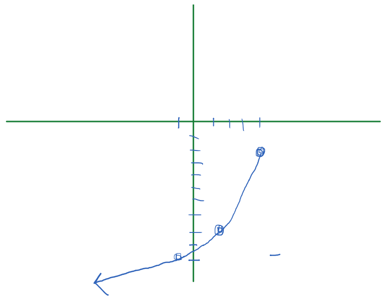
graph $\rightarrow$ $y = -2\sqrt{-3(x-4)} - 2$ $\rightarrow$ $y = -2\sqrt{3}\cdot\sqrt{-(x-4)} - 2$

h = 4, k = -2

x	y
✓ 1	-8
-1	-9.75

$a<0$
$b<0$

$x = 1$



1	-0
-1	-9.75

$x = 1$

$y = -2\sqrt{-3(1-4)} - 2$

$y = -2\sqrt{9} - 2$

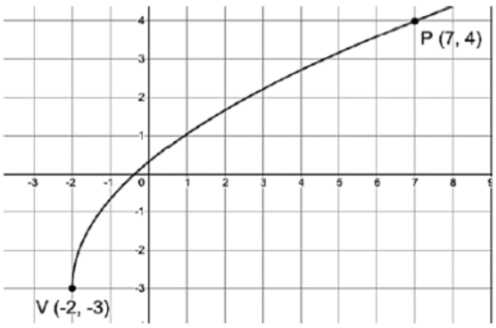
$y = -2(3) - 2$

$y = -8$

Finding the Rule

9) Find the rule for each square root function using the coordinates of the vertices, *V*, and the coordinates of point *P*.

a) $y = \frac{7}{3}\sqrt{x+2} - 3$



$y = a\sqrt{(x-h)} + k$

$y = a\sqrt{x+2} - 3$

$4 = a\sqrt{7+2} - 3$

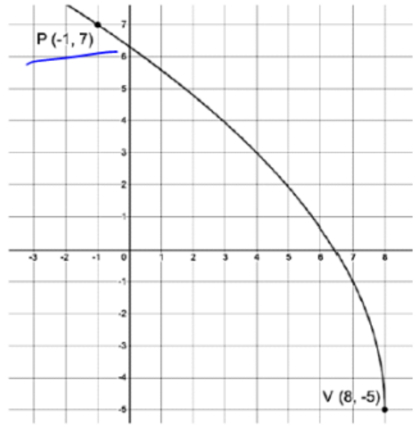
$7 = 3a \qquad a = \frac{7}{3}$

b)

$a > 0$
$b < 0$

$y = a\sqrt{-(x-h)} + k$

$y = a\sqrt{-(x-8)} - 5$



$7 = a\sqrt{-(-1-8)} - 5$

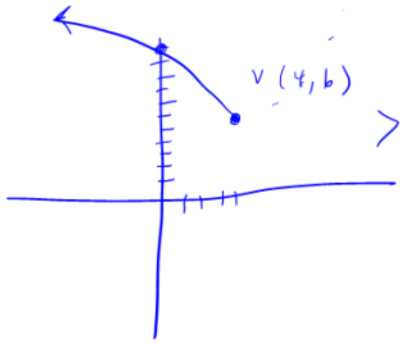
$7 = a\sqrt{9} - 5$

$12 = 3a$

$4 = a$

$y = 4\sqrt{-(x-8)} - 5$

10) Find the rule of a square root function with a vertex of (4, 6) and an initial value of 10.



$y = a\sqrt{-(x-h)} + k$

$10 = a\sqrt{-(0-4)} + 6$

$4 = 2a$

$2 = a$

$y = 2\sqrt{-(x-4)} + 6$

Solving Square Root Functions (Equalities)

- To solve for y, plug in x and solve.
- To solve for x,
  - Isolate the radical
  - State domain restrictions (the value under the radical cannot be negative)

- Solve for x
- Check for extraneous answers (plug solution into the equation)

Ex. Given $y = 2\sqrt{2x-4}$, solve when $x = 4$

$$y = 2\sqrt{2(4)-4}$$
$$y = 2\sqrt{4}$$
$$y = 4$$

Ex. Given $y=2\sqrt{2x-4}$, solve for x when $y=0$.

DOMAIN RESTRICTION

$$2x-4 \geq 0$$
$$2x \geq 4$$
$$x \geq 2$$

SOLVE

$$y = 2\sqrt{2x-4}$$
$$0 = 2\sqrt{2x-4}$$
$$0 = \sqrt{2x-4} \quad \text{(square both sides)}$$
$$0 = 2x-4$$
$$4 = 2x$$
$$2 = x$$

CHECK

$$y = 2\sqrt{2x-4}$$
$$0 = 2\sqrt{2(2)-4}$$
$$0 = 0 \quad \text{TRUE}$$

$$\underline{x=2}, \quad x \geq 2$$

Ex. Given $y = 2\sqrt{x-3}$, solve for x when $y = 4$

Domain Restriction

$$x-3 \geq 0$$
$$x \geq 3$$

Solve

$$4 = 2\sqrt{x-3}$$
$$(2)^2 = (\sqrt{x-3})^2$$
$$4 = x-3$$
$$7 = x$$

Check

$$4 = 2\sqrt{7-3}$$
$$4 = 2\sqrt{4}$$
$$4 = 4 \checkmark \quad \text{TRUE}$$

Ex. Given $y = 2\sqrt{x+3} + 2$, solve for the zero of the function
x int, $y = 0$

$$0 = 2\sqrt{x+3} + 2$$
$$-2 = 2\sqrt{x+3}$$
$$-1 = \sqrt{x+3}$$

Restriction

$$x+3 \geq 0$$
$$x \geq -3$$

- because this is not a positive root, there is no solution
- square root functions are the inverse of quadratic; we

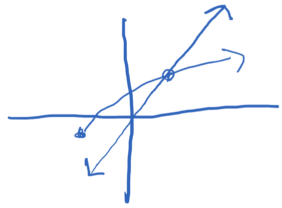
always use + root

Function Intersections

- use logic

$f(x) = 2\sqrt{x+4} - 1$

$g(x) = x$



Bad art diagram

Point of Intersection

$x = 2\sqrt{x+4} - 1$

$\left(\frac{x+1}{2}\right)^2 = \left(\sqrt{x+4}\right)^2$

$\frac{(x+1)(x+1)}{4} = x+4$

$x^2+2x+1 = 4(x+4)$

$x^2+2x+1 = 4x+16$

$* \; x^2-2x-15=0 \; *$

$(x+3)(x-5)=0$

$x=-3$

$x=5$

Domain Restriction

$x \geq -4$

Both answers are consistent with the restrictions.

Check for extraneous answers

$x=5$

$5 = 2\sqrt{5+4} - 1$

$5 = 2\sqrt{9} - 1$

$5 = 5$ TRUE

$x=-3$

$-3 = 2\sqrt{-3+4} - 1$

$-3 = 2(1) - 1$

$-3 = 1$

FALSE

$\therefore$ The functions intersect at the point $(5,5)$

Inequalities

Steps:
1. Isolate the radical
2. state domain restrictions
3. Solve
4. Check against domain restrictions

Ex. Solve $2\sqrt{2x+4} - 6 > 2$

$\sqrt{2x+4} > 4$

$2x+4 > 16$

$2x > 12$

$x > 6$

Domain Restriction

$2x+4 \geq 0$

$2x \geq -4$

$x \geq -2$

Check. $x > 6$ is consistent with DR.

$-\infty$ to ∞

Solution: $]6, \infty[$ or $x > 6$

Ex. $-2\sqrt{x-3}+4>0$

$\sqrt{x-3}<2$

$x-3<4$

$x<7$

Domain Restriction

$x-3\geq 0$

$x\geq 3$

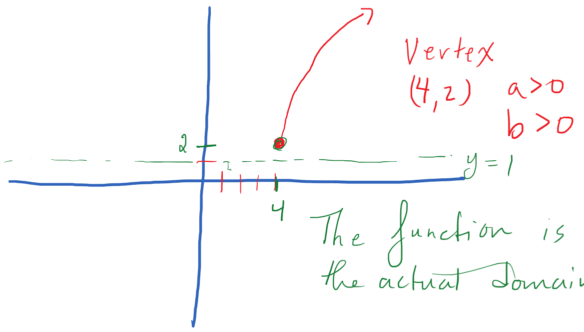
Solution: $3\leq x<7$

or $[3,7[$

Ex. Solve: $2\sqrt{3(x-4)}+2\geq 1$

$\sqrt{3(x-4)}\geq -0.5$

We know that the term under the radical cannot equal a negative, so we either have no solution or a solution that exists everywhere the function is defined.



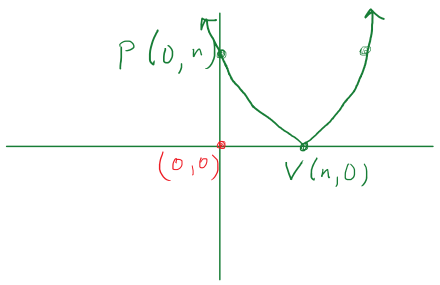
The function is always ≥ 1 but limited to the actual domain of the function (and ranges too)

$x\geq 4$ ✓ or $2\sqrt{3(x-4)}+2\geq 1$ over $[4,\infty[$

Pause: CONJECTURES

- Create 3 examples
- Use diverse numerical values for the basic unknowns of the question (make up numbers & sub-in)
- Write a conjecture statement based on the observed relationship

Ex. Consider a parabola with a vertex on the x-axis. The distance between the vertex and the origin is the same distance as the initial value and the origin. Formulate a conjecture describing the value of parameter "a".



Example 1	Example 2	Example 3
$\boxed{\text{Let } n=3}$	$\boxed{\text{Let } n=4}$	$\boxed{n=0.5}$
$y=a(x-3)^2+0$		
$3=a(0-3)^2+0$	$4=a(0-4)^2+0$	$0.5=a(0-0.5)^2+0$
$3=9a$	$4=16a$	$0.5=0.25a$
$\boxed{\frac{1}{3}=a}$	$\boxed{\frac{1}{4}=a}$	$\boxed{a=2}$

The value of "a" when the distance between the parabola vertex and

The value of "a" when the distance between the parabola vertex and the origin is the same distance as the initial value of the parabola and the origin, is

$$\frac{\pm 1}{\text{distance of vertex from origin}}$$

$a = \frac{1}{n}$

Finding the Inverse of a Square Root Function

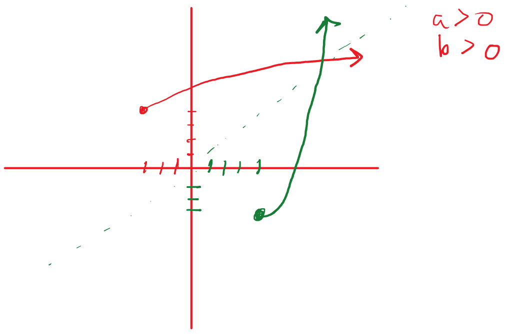
Recall: the domain of a function is the range of its inverse, and the range becomes the domain.

Ex. Find the inverse of $y = \sqrt{\frac{x+3}{2}} + 4$, sketch both, and state domain & range of both.

$y = \sqrt{\frac{1}{2}(x+3)} + 4$

Domain: $[-3, \infty[$

Range: $[4, \infty[$



Inverse

$x = \sqrt{\frac{1}{2}(y+3)} + 4$

$x - 4 = \sqrt{\frac{1}{2}(y+3)}$

$(x-4)^2 = \frac{1}{2}(y+3)$

$2(x-4)^2 = y + 3$

$2(x-4)^2 - 3 = y$

Note the restricted domain, which gives us half the parabola.

Domain: $[4, \infty[$

Range: $[-3, \infty[$

Take the positive root!